APPLICATION OF THE PARAMETRIC METHOD TO THE SOLUTION OF PROBLEMS OF THE UNSTEADY THERMAL BOUNDARY LAYER

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A generalization of a parametric method developed earlier for the calculation of the dynamic boundary layers is presented for problems of the unsteady laminar thermal boundary layer.

A generalization of the parametric method of Loitsyanskii [1] to problems of the thermal boundary layer was accomplished by Duric [2]. For a solution of the equation of the thermal boundary layer the author used a parametric method which he developed earlier for calculating the unsteady dynamic layer with a velocity at the outer boundary which varies according to the law $U(x, t) = v(x)\lambda(t)$. In [3] this same method is used to study the thermal boundary layer near the leading critical point of a heated cylinder set into motion by an impulse, with the results of the calculation being in good agreement with data obtained by a method of successive approximations [4] similar to the Blasius method for the dynamic problem.

In the present work the results of the parametric method presented in [5] are used to solve the equation of the thermal unsteady laminar boundary layer in an incompressible fluid.

1. The equation of motion of a flat unsteady laminar boundary layer in an incompressible fluid for a stream function ψ has the form

$$\begin{gathered}\frac{\partial^2\psi}{\partial t\partial y}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\dot{U}+UU'+\nu\frac{\partial^3\psi}{\partial y^3},\\ \psi=\frac{\partial\psi}{\partial y}=0 \text{ at } y=0,\ \frac{\partial\psi}{\partial y}\to U(x, t) \text{ at } y=\infty,\\ \frac{\partial\psi}{\partial y}=u_0(x, y) \text{ at } t=t_0,\ \frac{\partial\psi}{\partial y}=u_1(t, y) \text{ at } x=x_0.\end{gathered} \tag{1}$$

Here and afterward the prime and the dot above a letter denote partial derivatives with respect to x and t, respectively.

If it is required to determine the temperature field in the fluid, then to the system (1) is added the energy equation, which in the given case can be written as follows:

$$\frac{\partial T}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\frac{\nu}{\mathrm{Pr}}\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+\frac{1}{c_p}(\dot{U}+UU')\left(U-\frac{\partial\psi}{\partial y}\right) \tag{2}$$

with the boundary conditions

$$\begin{gathered}\text{a) } T=T_w(x, t) \text{ or b) } \frac{\partial T}{\partial y}=q_w(x, t) \text{ at } y=0,\ T=T_\infty \text{ at } y=\infty,\\ T=T_0(x, y) \text{ at } t=t_0,\ T=T_1(t, y) \text{ at } x=x_0.\end{gathered} \tag{3}$$

The condition a) is used in the present article, i.e., Eq. (2) is solved for the case when the wall temperature T_W is given, although the solution of the problem with the heat flux q_W given — condition b) — can be obtained in a similar way.

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The last two terms in (2), which have the order of the Eckert number ($Ec = U_\alpha^2/c_p\Delta T$), take into account the heat produced near the surface of the body through viscous dissipation and compression, respectively. It must be noted that this formulation of the problem, when the physical parameters of the fluid are assumed to be constant, is valid only in a very narrow range of temperature drops $\Delta T = T_W - T_\infty$ and at small velocities U_∞ of the incoming stream. For example, for air under standard conditions one can take ν = const, Pr = const, and ρ = const with an error of 5% only within the limits of $\Delta T \leq 15°$ and $U_\infty \leq 100$ m/sec, and for liquids the range of ΔT is even smaller because of the sharp temperature dependence of ν and Pr. For the latter the interval ΔT can be widened somewhat, however, if in the equations one uses the values of the physical parameters taken at some mean determining temperature T^* out of the given interval ΔT. With such a temperature drop and in the case of moderate velocities 50 m/sec $\leq U_\infty \leq 100$ m/sec for the incoming stream the Eckert number varies in the range of $2/3 \geq Ec \geq 1/6$, and therefore, it becomes necessary to allow for the last two terms in Eq. (2).

2. First let us consider the transformation of the equation (1) for the dynamic boundary layer, which is possible because of its self-similarity. Let us write the integral momentum equation for the unsteady boundary layer. We have

$$\frac{1}{U}\frac{\partial\delta^*}{\partial t}+\frac{\dot{U}}{U^2}\delta^*+\frac{\partial\delta^{**}}{\partial x}+\frac{U'}{U}(2\delta^{**}+\delta^*)=\frac{\tau_w}{\rho U^2}, \quad (4)$$

where

$$\delta^*(x,t)=\int_0^\infty\left(1-\frac{u}{U}\right)dy;\ \delta^{**}(x,t)=\int_0^\infty\frac{u}{U}\left(1-\frac{u}{U}\right)dy;\ \tau_w(x,t)=\mu\left.\frac{\partial u}{\partial y}\right|_{y=0}.$$

We introduce the dimensionless characteristic functions

$$H^*(x,t)=\int_0^\infty\left(1-\frac{u}{U}\right)d\left(\frac{y}{h}\right),\ H^{**}(x,t)=\int_0^\infty\frac{u}{U}\left(1-\frac{u}{U}\right)d\left(\frac{y}{h}\right),\ \zeta(x,t)=\left[\frac{\partial(u/U)}{\partial(y/h)}\right]_{y=0}, \quad (5)$$

where $h(x, t)$ is some characteristic linear scale of the transverse coordinate in the boundary layer. Then with the help of the new dimensionless variables

$$\varphi(\eta, f_{kn}, g)=\frac{B\psi(x,y,t)}{U(x,t)h(x,t)};\ \eta=\frac{By}{h(x,t)}, \quad (6)$$

as well as the infinite series of parameters

$$f_{kn}=U^{k-1}\frac{\partial^{k+n}U}{\partial x^k\,\partial t^n}z^{k+n}\ (k,\ n=0,\ 1,\ 2,\ \ldots) \quad (7)$$

($z = h^2/\nu$) and the constant parameter

$$g=\dot{z}=\text{const} \quad (8)$$

Eq. (1) is reduced with the use of (4) to the "universal" form, universal in the sense that neither the equation itself nor its boundary conditions will depend explicitly on $U(x, t)$.

We can write the "universal" equation in the form

$$B^2\frac{\partial^3\varphi}{\partial\eta^3}+f_{10}\left[\varphi\frac{\partial^2\varphi}{\partial\eta^2}-\left(\frac{\partial\varphi}{\partial\eta}\right)^2+1\right]+f_{01}\left(1-\frac{\partial\varphi}{\partial\eta}\right)+\frac{1}{2}\frac{\partial^2\varphi}{\partial\eta^2}[\varphi F+\eta g]=\sum_{k,n=0}^{\infty}\left[E\frac{\partial^2\varphi}{\partial f_{kn}\partial\eta}+D(f_{kn},g)\right.$$
$$\left.\times\left(\frac{\partial\varphi}{\partial\eta}\frac{\partial^2\varphi}{\partial f_{kn}\partial\eta}-\frac{\partial^2\varphi}{\partial\eta^2}\frac{\partial\varphi}{\partial f_{kn}}\right)\right], \quad (9)$$
$$\varphi=\frac{\partial\varphi}{\partial\eta}=0\ \text{at}\ \eta=0,\ \frac{\partial\varphi}{\partial\eta}\to 1\ \text{at}\ \eta=\infty,$$
$$\varphi=\varphi_0(\eta)\ \text{at}\ g=f_{kn}=0\ (k,\ n=0,\ 1,\ 2,\ \ldots),$$

where $\varphi_0(\eta)$ is the Blasius solution for the steady boundary layer at a plate;

$$E(f_{kn},g)=[(k-1)f_{01}f_{kn}+(k+n)f_{kn}g+f_{k,n+1}],\ D(f_{kn},g)=[(k-1)f_{10}f_{kn}+(k+n)f_{kn}F+f_{k+1,n}], \quad (10)$$

while the function $F(f_{kn}, g)$ is determined from the equation

$$F=\left[\zeta-f_{10}(2H^{**}+H^{*})-(f_{01}+g/2)H^{**}-\right.$$

$$\left.-\sum_{k,n=0}^{\infty}\left\{E\frac{\partial H^{*}}{\partial f_{kn}}+[(k-1)f_{10}f_{kn}+f_{k+1,n}]\frac{\partial H^{*}}{\partial f_{kn}}\right\}\right]\times$$

$$\times\left[\frac{H^{**}}{2}+\sum_{k,n=0}^{\infty}\frac{\partial H^{**}}{\partial f_{kn}}(k+n)f_{kn}\right]^{-1}, \tag{11}$$

which is obtained by a transformation of Eq. (4).

We note that Eq. (7) is exact for a broad class of velocities $U(x, t)$ for which $z = At + C(x)$, where A is an arbitrary constant, and $C(x)$ is some function of the longitudinal coordinate.

3. For the solution of the problem of the thermal boundary layer we introduce into the discussion the dimensionless temperature θ:

$$\theta(x, y, t)=\frac{T-T_{\infty}}{T_w-T_{\infty}} \tag{12}$$

and as a supplement to the parameters (7) and (8) the infinite series of parameters

$$l_{ij}=\frac{U^i}{\vartheta}\frac{\partial^{i+j}\vartheta}{\partial x^i\partial t^j}z^{i+j}\quad (i, j=0, 1, 2, \ldots), \tag{13}$$

where $\vartheta = T_W - T_\infty$. In particular, the first parameters of this series will be

$$l_{10}=\frac{U}{\vartheta}\vartheta' z;\quad l_{01}=\frac{\dot{\vartheta}}{\vartheta}z. \tag{14}$$

We note that a corresponding value characterizing the dynamic layer ($z = z_d$) is used as the transverse scale of the thermal boundary layer. This means that the previous history, expressed by the last two equalities in (3), is not taken into account. Thus, if in the problem one is required to study the development of the temperature profile given at some "initial" cross section of the layer and the temperature field in it at the starting time, then because of the parabolic nature of the boundary-layer equation a solution in parametric form is possible only at a certain distance from the "initial" cross section and from the starting time.

We will seek a solution for Eq. (2) with the corresponding boundary conditions from (3) in the following form:

$$T=T_{\infty}+(T_w-T_{\infty})\theta_1(\eta, f_{kn}, l_{ij}, g)+\frac{U^2}{c_p}\theta_2(\eta, f_{kn}, g). \tag{15}$$

Then, changing in Eq. (2) to the dimensionless functions φ and θ and the new variables η, f_{kn}, and l_{ij} and the value g, which plays the role of a constant parameter, and in a corresponding way separating the equations and boundary conditions obtained in the course of the transformation, we find the following equations for θ_1 and θ_2:

$$\frac{B^2}{\Pr}\frac{\partial^2\theta_1}{\partial\eta^2}+\left[\left(f_{10}+\frac{1}{2}F\right)\varphi+\frac{1}{2}g\eta\right]\frac{\partial\theta_1}{\partial\eta}-\left(l_{01}+l_{10}\frac{\partial\varphi}{\partial\eta}\right)\theta_1=$$

$$=\sum_{k,n=0}^{\infty}\left(E_{kn}\frac{\partial\theta_1}{\partial f_{kn}}+D_{kn}\frac{\partial\theta_1}{\partial f_{kn}}\frac{\partial\varphi}{\partial\eta}-D_{kn}\frac{\partial\theta_1}{\partial\eta}\frac{\partial\varphi}{\partial f_{kn}}\right)+\sum_{i,j=0}^{\infty}\left(L_{ij}\frac{\partial\theta_1}{\partial l_{ij}}+N_{ij}\frac{\partial\varphi}{\partial\eta}\frac{\partial\theta_1}{\partial l_{ij}}\right), \tag{16}$$

$$\theta_1=1 \text{ at } \eta=0,\ \theta_1=0 \text{ at } \eta=\infty\ \theta_1=\theta_{10}(\eta, g) \text{ at } l_{ij}=f_{kn}=0\quad (k, n, i, j=0, 1, 2, \ldots), \tag{17}$$

where $\theta_{10}(\eta, g)$ is the solution of the equation when $l_{ij} = f_{kn} = 0$;

$$\frac{B^2}{\Pr}\frac{\partial^2\theta_2}{\partial\eta^2}+\left[\left(f_{10}+\frac{1}{2}F\right)\varphi+\frac{1}{2}\eta g\right]\frac{\partial\theta_2}{\partial\eta}-2\left(f_{01}+f_{10}\frac{\partial\varphi}{\partial\eta}\right)\theta_2=$$

$$=\sum_{k,n=0}^{\infty}\left(E_{kn}\frac{\partial\theta_2}{\partial f_{kn}}-\frac{\partial\theta_2}{\partial\eta}\frac{\partial\varphi}{\partial f_{kn}}D_{kn}+\frac{\partial\varphi}{\partial\eta}\frac{\partial\theta_2}{\partial f_{kn}}D_{kn}\right)$$

$$-B^2\left(\frac{\partial^2\varphi}{\partial\eta^2}\right)^2-(f_{10}+f_{01})\left(1-\frac{\partial\varphi}{\partial\eta}\right), \tag{18}$$

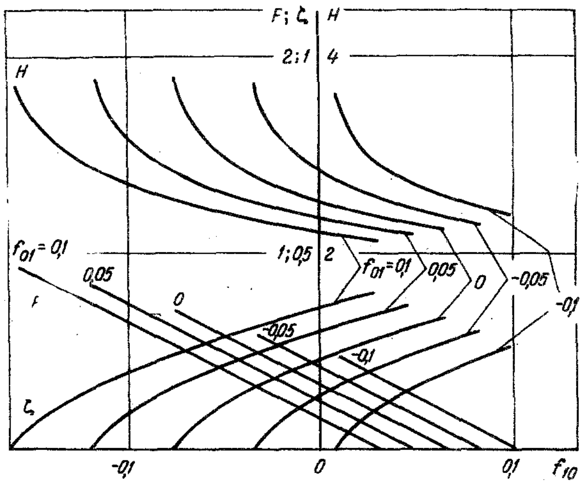

Fig. 1. Dependence of characteristic functions of the dynamic boundary layer on the parameter f_{10}.

$$\theta_2 = 0 \text{ at } \eta = 0, \ \theta_2 = 0 \text{ as } \eta \to \infty,$$

$$\theta_2 = \theta_{20}(\eta, g) \text{ at } f_{kn} = 0 \quad (k, n = 0, 1, 2, \ldots), \tag{19}$$

where $\theta_{20}(\eta, g)$ is the solution of the equation at $f_{kn} = 0$.

For L_{ij} and N_{ij} we have the expressions

$$L_{ij} = [if_{01} - l_{01} + (i + j) g] l_{ij} + l_{i,j+1},$$
$$N_{ij} = [if_{10} - l_{10} + (i + j) F] l_{ij} + l_{i+1,j}.$$

The equations and boundary conditions (16)-(19) do not depend explicitly on the form of the functions $T_w(x, t)$ and $U(x, t)$, and in this sense they can be called "universal" just like Eq. (9).

For the determination of the heat flux at the wall we write the equality

$$q_w = -\lambda \frac{\partial T}{\partial y}\bigg|_{y=0} = -\lambda \frac{\vartheta}{h}\left(B \frac{\partial \theta}{\partial \eta}\bigg|_{\eta=0}\right) = -\lambda \frac{\vartheta}{h} \zeta_T = -\lambda \frac{\vartheta}{h}\left(\zeta_{T1} + \frac{U^2}{c_p \vartheta} \zeta_{T2}\right), \tag{20}$$

where

$$\zeta_{Ti} = B \frac{\partial \theta_i}{\partial \eta}\bigg|_{\eta=0} \quad (i = 0, 1, 2).$$

Let us introduce into the discussion the Nusselt number

$$\mathrm{Nu}_h = \frac{\alpha(x, t) h(x, t)}{\lambda} = \frac{q_w h}{\lambda \Delta T} = -\zeta_{T1}, \tag{21}$$

where

$$\Delta T = T_w - T_\infty + \frac{U^2}{c_p} \frac{\zeta_{T2}}{\zeta_{T1}} \quad (\zeta_{T1} \neq 0). \tag{22}$$

As a result we have the following expression for the heat flux at the wall:

$$q_w = \lambda \frac{\Delta T(x, t)}{h(x, t)} \mathrm{Nu}_h. \tag{23}$$

Equations (9), (16), and (18) are integrated once and for all for a fixed value of the Prandtl number in the m-parametric approximation. The characteristic functions obtained in this case can be used for the approximate solution of problems having an arbitrarily assigned velocity $U(x, t)$ and temperature $T_w(x, t)$ expressed through sufficiently smooth functions. Before the integration of the equations it is necessary to choose some characteristic value as the scale $h(x, t)$ of the transverse coordinate in the boundary layer. It is convenient to choose $h = \delta^{**}$, where $H^{**} = 1$, $H^* = \delta^*/\delta^{**} = H$, and Eq. (11) takes the form

$$F = Uz' = 2\left[\zeta - f_{10}(2 + H) - \left(f_{01} + \frac{g}{2}\right) H - \sum_{k,n=0}^{\infty} E_{kn} \frac{\partial H}{\partial f_{kn}}\right]. \tag{24}$$

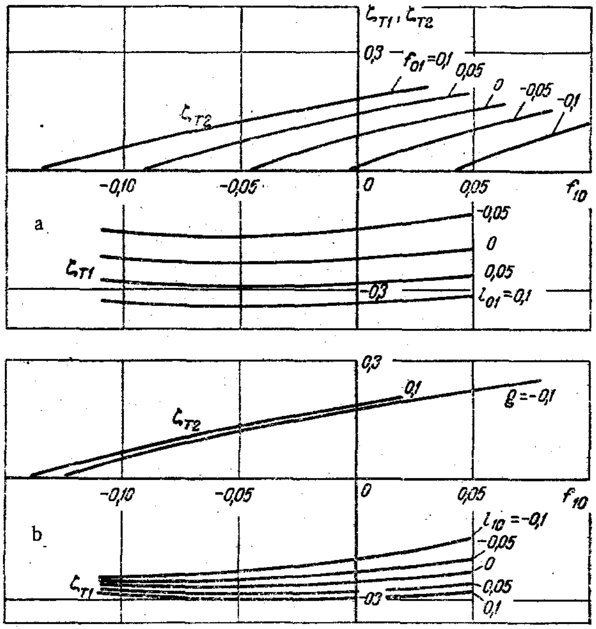

Fig. 2. Dependence of the reduced coefficients ζ_{T1} and ζ_{T2} of the heat fluxes at the wall at Pr = 0.72 on the parameter f_{10}: a) when l_{01} = var and $l_{10} = f_{01} = g = 0.05$ for ζ_{T1}; f_{01} = var and $l_{10} = l_{01} = g = 0.05$ for ζ_{T2}; b) when l_{10} = var and $l_{01} = f_{01} = g = 0.05$ for ζ_{T1}; g = var and $l_{10} = l_{01} = f_{01} = 0.05$ for ζ_{T2}.

4. Equations (9), (16), and (18) were solved in a local approximation in which the parameters f_{10}, f_{01}, g, l_{10}, and l_{01} were retained and all the rest of the parameters and the derivatives with respect to the parameters were discarded. In this case the indicated equations take the form

$$B^2\frac{d^3\varphi}{d\eta^3}+\frac{(F+2f_{10})\varphi+\eta g}{2}\frac{d^2\varphi}{d\eta^2}+f_{10}\left[1-\left(\frac{d\varphi}{d\eta}\right)^2\right]+f_{01}\left(1-\frac{d\varphi}{d\eta}\right)=0,$$
$$\varphi=\frac{d\varphi}{d\eta}=0 \text{ at } \eta=0,\ \frac{d\varphi}{d\eta}\to 1 \text{ as } \eta\to\infty,$$
$$\varphi=\varphi_0(\eta) \text{ at } f_{10}=f_{01}=g=0, \tag{25}$$

where $\varphi_0(\eta)$ is the Blasius solution for the steady boundary layer at a plate and B = 0.47;

$$\frac{B^2}{\Pr}\frac{d^2\theta_1}{d\eta^2}+\left[\left(f_{10}+\frac{1}{2}F\right)\varphi+\frac{1}{2}g\eta\right]\frac{d\theta_1}{d\eta}-\left(l_{01}+l_{10}\frac{d\varphi}{d\eta}\right)\theta_1=0, \tag{26}$$
$$\theta_1=1 \text{ at } \eta=0,\ \theta_1=0 \text{ as } \eta\to\infty.$$

Here θ_1 is the solution of the problem of cooling of the wall;

$$\frac{B^2}{\Pr}\frac{d^2\theta_2}{d\eta^2}+\left[\left(f_{10}+\frac{1}{2}F\right)\varphi+\frac{1}{2}\eta g\right]\frac{d\theta_2}{d\eta}-2\left(f_{01}+f_{10}\frac{d\varphi}{d\eta}\right)\theta_2$$
$$=-B^2\left(\frac{d^2\varphi}{d\eta^2}\right)^2-(f_{10}+f_{01})\left(1-\frac{d\varphi}{d\eta}\right), \tag{27}$$
$$\theta_2=0 \text{ at } \eta=0,\ \theta_2=0 \text{ as } \eta\to\infty,$$

where θ_2 is the solution of the problem of the dissipation of mechanical energy in the boundary layer through friction and compression. In the approximation adopted the functional F in the equations is calculated from the equation

$$F(f_{10}, f_{01}, g)=Uz'=2\left[\zeta-f_{10}(2+H)-\left(f_{01}+\frac{g}{2}\right)H\right]. \tag{28}$$

The system of equations (25)-(28) was integrated for the values Pr = 0.72 and Pr = 1 on a BESM-2 computer by the trial-run method with iterations. Graphs of the characteristic functions were constructed as a result (Figs. 1 and 2). Curves of the dependence of the characteristic functions ζ, F, and H on the parameter f_{10} for a number of values of the unsteadiness parameter f_{01} and at a fixed value of the parameter $g = 0.05$ are shown in Fig. 1. In the range of variation of the parameters of $-0.1 \le f_{01} \le 0.1$ and $-0.2 \le g \le 0.2$ and of f_{10} from the value at the leading critical point to the value at the separation point the functional F can be approximated with a sufficient degree of accuracy by the linear dependence

$$F = 0.44 - 5.3f_{10} - 1.65f_{01} - 2.1g. \tag{29}$$

The function ζ_{T2} decreases upon approach to the separation of the boundary layer and upon slowing of the stream (Fig. 2a). These effects can easily be explained if one recalls that ζ_{T2} depends on the dissipation of mechanical energy in the boundary layer. The effect on ζ_{T2} of the parameter g is insignificant (Fig. 2b) and shows up noticeably only in the divergent region of the boundary layer. The function ζ_{T1} depends weakly on the parameters g, f_{01}, and l_{10} (Fig. 2b). The effect of f_{10} shows up noticeably only in the convergent region of the boundary layer, with the absolute value of ζ_{T1} decreasing with an increase in f_{10} (Fig. 2). In the range of variation of the parameters f_{01}, f_{10}, and g indicated above and in the limits of variation of $-0.1 \le l_{10} \le 0.1$ and $-0.1 \le l_{01} \le 0.1$ the functions ζ_{T1} and ζ_{T2} can be represented by the following dependences for $Pr = 0.72$:

$$\zeta_{T1} = -0.196 - 1.58l_{01} - 0.575l_{10} - 0.1g + 0.55f_{10} + 0.25f_{01}, \tag{30}$$

$$\zeta_{T2} = 0.085 + 1.5f_{10} + 1.2f_{01} + 0.05g, \tag{31}$$

where in the present approximation the next to last and the last terms in Eq. (30) are taken into account only in the convergent section of the boundary layer and in the presence of accelerating streams, respectively.

5. The results of the calculations were used to solve two particular problems: on the boundary layer at an infinite plane and near the leading critical point of a round cylinder during motion by an impulse.

a) Impulse motion of an infinite plane is characterized by the conditions

$$U(t) = \begin{cases} U_0 \text{ when } t > 0, \\ 0 \text{ at } t = 0, \end{cases} \quad T|_{y=0} = \begin{cases} T_w \text{ when } t > 0, \\ T_\infty \text{ at } t = 0. \end{cases}$$

Then $f_{10} = f_{01} = 0$, and from (29), seeing that $F = 0$, we obtain $g = \dot{z} = 0.21$, while for $z = \delta^{**2}/\nu$ we find $\delta^{**} = 0.457\sqrt{\nu t}$. In the case when $Pr = 0.72$ and $\Delta T = T_w - T_\infty = \text{const}$, using (20), (30), and (31) we obtain an expression for the heat flux at the wall,

$$q_w = 0.475\lambda \frac{\Delta T}{\sqrt{\nu t}}\left(1 - 0.438\,\frac{U_0^2}{c_p \Delta T}\right).$$

The exact solution [4] gives

$$q_w = 0.479\lambda \frac{\Delta T}{\sqrt{\nu t}}\left(1 - 0.443\,\frac{U_0^2}{c_p \Delta T}\right).$$

With the help of additional calculations not presented in the article an expression was obtained for the heat flux at the wall for the values $0.6 \le Pr \le 1.1$:

$$q_w = 0.560\lambda \frac{\Delta T}{\sqrt{\nu t}}\left(1 - 0.5\,\frac{U_0^2}{c_p \Delta T}\, Pr^{0.36}\right) Pr^{0.5}.$$

b) The impulse motion near the leading critical point of a round cylinder is characterized by the following conditions:

$$U(x, t) = \begin{cases} U_0 \bar{x} \text{ when } t > 0, \\ 0 \text{ at } t = 0, \end{cases} \quad T|_{y=0} = \begin{cases} T_w \text{ when } t > 0, \\ T_\infty \text{ at } t = 0. \end{cases}$$

Using Eqs. (20) and (29), as well as approximations of the characteristic functions ζ_{T1} and ζ_{T2} obtained from graphs constructed for the case of $Pr = 1$ and $\Delta T = \text{const}$, we find the expression for the Nusselt number,

$$\frac{Nu_x}{\sqrt{Re_x}} = \frac{q_w x}{\lambda \Delta T}\sqrt{\frac{\nu R}{U_0 x^2}} = 3.48\,[1 - \exp(-2.55\bar{t})]^{-\frac{1}{2}} \times \tag{32}$$

$$\times [0.1716 - Ec_0 \bar{x}^2 0.249 + \exp(-2.55\bar{t})(0.0934 + Ec_0 \bar{x}^2 0.118)],$$

where $Ec_0 = U_0^2/c_p\Delta T$; $\bar{t} = U_0 t/R$; $\bar{x} = x/R$. Then as $\bar{t} \to \infty$ and, neglecting the heat produced through friction and compression, $Nu_x Re_x^{-1/2} = 0.596$ as compared with the value 0.570 found from the exact solution

([6], p. 286). If in Eq. (32) the square bracket is equated to zero, then one determines the moment when the heat flux changes sign at a certain place on the surface:

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = 0 \text{ at the point } \bar{x} = \sqrt{\frac{2.0}{\mathrm{Ec}_0}} \text{ at the time } \bar{t} = 0,$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = 0 \text{ at the point } \bar{x} = \sqrt{\frac{0{,}69}{\mathrm{Ec}_0}} \text{ at the time } \bar{t} = \infty.$$

Thus, the heat flux changes sign in the course of time within the interval

$$\sqrt{\frac{0.69}{\mathrm{Ec}_0}} \leqslant \bar{x} \leqslant \sqrt{\frac{2.0}{\mathrm{Ec}_0}}. \tag{33}$$

From the exact solution [4] it was found that

$$\sqrt{\frac{0.274}{\mathrm{Ec}_0}} \leqslant \bar{x} \leqslant \sqrt{\frac{2.0}{\mathrm{Ec}_0}}. \tag{34}$$

We note that the solution of [4] is valid only for short times, and therefore, one can assert that only the upper limit of the inequality (34) is obtained with a sufficient degree of accuracy.

NOTATION

x, y	are the longitudinal and transverse coordinates in the boundary layer;
η	is the dimensionless transverse coordinate;
U and T_∞	are the velocity and temperature at the outer boundary of the boundary layer;
ψ	is the stream function;
T	is the temperature;
T_w	is the wall temperature;
φ	is the dimensionless stream function;
θ	is the dimensionless temperature;
u, v	are the projections of velocity in the boundary layer onto the x and y axes, respectively;
q	is the heat flux;
ρ	is the fluid density;
μ, ν	are the dynamic and kinematic viscosity coefficients;
Pr, Nu, Ec	are the Prandtl, Nusselt, and Eckert numbers, respectively;
t	is the time;
c_p	is the heat capacity at constant pressure;
h	is the scale of transverse coordinate in the boundary layer;
$z = h^2/\nu$;	
F, H*, H**, and H	are the characteristic functions;
δ^*	is the displacement thickness;
δ^{**}	is the thickness of momentum loss;
τ_w	is the surface friction stress;
ζ	is the reduced coefficient of friction;
ζ_{T1}, ζ_{T2}	are the reduced heat-flux coefficients;
B	is the normalizing factor;
f_{kn}, g, l_{ij}	are the dimensionless parameters;
R	is the radius of the cylinder;
U_0	is the velocity of incoming flow.

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